INTEGRATION

Integration is a process, which is a inverse of differentiation. As the symbol $\frac{d}{dx}$ represents differentiation with respect to x, the symbol $\int dx$ stands for integration with respect to x.

Definition

If $\frac{d}{dx}\left[f(x)\right]=F(x)$ then f(x) is called the integral of F(x) denoted by $\int F(x)dx = f(x)+c$. This can be read it as integral of F(x) with respect to x is f(x) + c where c is an arbitrary constant. The integral $\int F(x)dx$ is known as **Indefinite integral and the function F(x) as integrand.**

Formula on integration

1). $\int x^n dx = \frac{x^{n+1}}{n+1}$ **+c** (n ≠-1)

2). $\int \frac{1}{x} dx = \log x$ **+c**

3). $\int dx$ **= x+c**

4). $\int a^x dx = \frac{a^x}{\log a}$ **+c**

5). $\int e^x$ dx = e^x **+c**

6). $\int (u(x)+v(x))dx = \int u(x)dx + \int v(x)dx$

7). $\int (c_1u(x) \pm c_2v(x))dx = \int c_1u(x)dx \pm \int c_2v(x)dx$

8). $\int cdx$ **=** c x + d

9). $\int \sin x\, dx = -\cos x$ **+c**

10). $\int \cos x\, dx = \sin x$ **+c**

11). $\int \sec^2 x dx = \tan x$ **+c**

12). $\int \cos ec^2 x dx = -\cot x$ **+c**

13). $\int \sec x \tan x \, dx = \sec x + c$

14). $\int \cos ecx \cot x \, dx = -\cos ec \, x + c$

13). $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$ **+c**

14). $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \frac{a + x}{a - x}$ **+c**

15). $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \frac{x - a}{x + a}$ **+c**

16). $\int \frac{1}{1 - x^2} dx = \tan^{-1} x + c$

Definite integral

If f(x) is indefinite integral of F(x) with respect to x then the Integral $\int_a^b F(x) dx$ is called definite integral of F(x) with respect to x from x = a to x = b. Here a is called the Lower limit and b is called the Upper limit of the integral.

$\int_a^b F(x) dx$ = $[f(x)]_a^b$ = f(Upper limit) - f(Lower limit)

= f(b) - f(a)

Note

While evaluating a definite integral no constant of integration is to be added. That is a definite integral has a definite value.

Method of substitution

Method –1

Formulae for the functions involving (ax + b)

Consider the integral

I = $\int (ax+b)^n dx$ -------------(1)

Where a and b are constants

Put a x + b = y

Differentiating with respect to x

a dx + 0 = dy

$$dx = \frac{dy}{a}$$

Substituting in (1)

I = $\int y^n . \frac{1}{a} dy$ +c

= $\frac{1}{a}\int y^n .dy$ +c

= $\frac{1}{a}\frac{y^{n+1}}{n+1}$ +c

= $\frac{1}{a}\left[\frac{(ax+b)^{n+1}}{n+1}\right]$ + c

Similarly this method can be applied for other formulae also.

Method II

Integrals of the functions of the form

$$\int f(x^n) x^{n-1}\, dx$$

put x^n =y,

$$nx^{n-1} = \frac{dy}{dx}$$

$$x^{n-1}dx = \frac{dy}{n}$$

Substituting we get

I = $\int f(y)\frac{dy}{n}$ and this can be integrated.

Method –III

Integrals of function of the type

$$\int [f(x)]^n f^1(x) dx$$

when n ≠ -1, put f(x) = y then $f^1(x)dx = dy$

$$\therefore \int [f(x)]^n f^1(x) dx = \int y^n \, dy$$

$$= \frac{y^{n+1}}{n+1}$$

$$= \frac{[f(x)]^{n+1}}{n+1}$$

when n= -1, the integral reduces to

$$\frac{f^1(x)}{f(x)} dx$$

putting y = f(x) then dy = f^1(x) dx

$$\therefore \int \frac{dy}{y} = \log y = \log f(x)$$

Method IV

Method of Partial Fractions

Integrals of the form $\int \frac{dx}{ax^2 + bx + c}$

Case.1

If denominator can be factorized into linear factors then we write the integrand as the sum or difference of two linear factors of the form

$$\frac{1}{(ax^2 + bx + c} = \frac{1}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

Case-2

In the given integral $\int \frac{dx}{ax^2 + bx + c}$ the denominator $ax^2 + bx + c$ can not be factorized into linear factors, then express $ax^2 + bx + c$ as the sum or difference of two perfect squares and then apply the formulae

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \frac{a+x}{a-x}$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \frac{x-a}{x+a}$$

Integrals of the form $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$

Write denominator as the sum or difference of two perfect squares

$$\int \frac{dx}{\sqrt{ax^2 + bx + c}} = \int \frac{dx}{\sqrt{x^2 + a^2}} \quad \textbf{or} \int \frac{dx}{\sqrt{x^2 - a^2}} \quad \textbf{or} \int \frac{dx}{\sqrt{a^2 - x^2}}$$

and then apply the formula

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \log(x + \sqrt{x^2 + a^2})$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log(x + \sqrt{x^2 - a^2})$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$$

Integration by parts

If the given integral is of the form $\int udv$ then this can not be solved by any of techniques studied so far. To solve this integral we first take the product rule on differentiation

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Integrating both sides we get

$$\int \frac{d(uv)}{dx} dx = \int \left(u\frac{dv}{dx} + v\frac{du}{dx}\right)dx$$

then we have $uv = \int vdu + \int udv$

re arranging the terms we get

$\int udv = uv - \int vdu$ This formula is known **as integration by parts formula**

Select the functions u and dv appropriately in such a way that integral $\int vdu$ can be more easily integrable than the given integral

APPLICATION OF INTEGRATION

The area bounded by the function y=f(x), x=axis and the ordinates at x=a x=b is given by $A = \int_a^b f(x)dx$