Lecture No.9

Expansion path, ridgeline and least cost combination of inputs

vii) Least cost combination: The problem here is to find out a combination of inputs, which should cost the least, i.e., minimization of cost. The tangency of isocost and isoquant would indicate the least cost combination of X_1 and X_2, i.e., slope of isoquant = slope of isocost. Least cost combination is given, algebraically, by equating

$$MRS_{x2\,x1} = \frac{\Delta X_1}{\Delta X_2} = -\frac{Px_2}{Px_1} \text{, i.e., } -Px_1(\Delta X_1) = Px_2(\Delta X_2)$$

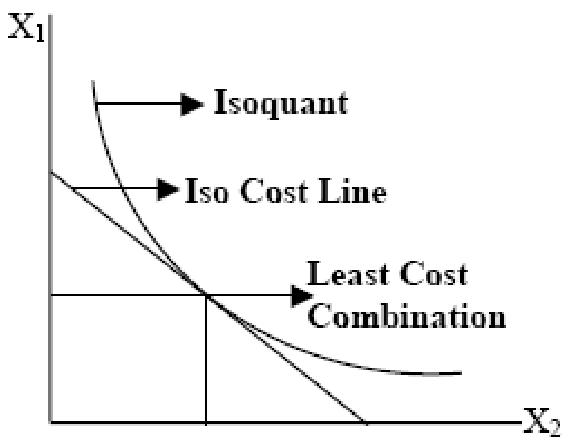


Fig.11.5 Least Cost Combination

If $-Px_1(\Delta X_1) > Px_2(\Delta X_2)$, then, thecost of producing the given output amountcould be reduced by increasing X_2anddecreasing X_1because the cost of an addedunit of $X_2$is less than the cost of the unitsof X_1, it replaces. On the other hand, ifbetween two points of the isoquant,$Px_1(\Delta X_1) < -Px_2(\Delta X_2)$, then the cost of producing the specified quantity of output can be reduced by using less X_2 and adding X_1The marginal physical product equations can be used to determine the returns per rupee spent at the least cost point. Rewriting the least cost combination as:

$$\frac{MPPx_1}{Px_1} = \frac{MPPx_2}{Px_2}$$

$$-\frac{\Delta X_1}{\Delta X_2} = \frac{dX_1}{dX_2} = \frac{MPPx_2}{MPPx_1} = \frac{Px_2}{Px_1}$$

$$Y = f(X_1, X_2)$$

Total differential is:

$$dY = \frac{\delta Y}{\delta X_1} dX_1 + \frac{\delta Y}{\delta X_2} dX_2 = 0$$

$$-\frac{dX_1}{dX_2} = \text{MRS (or) RTS} = \frac{(\delta Y/\delta X_2)}{(\delta Y/\delta X_1)} = -\frac{MPPx_2}{MPPx_1}$$

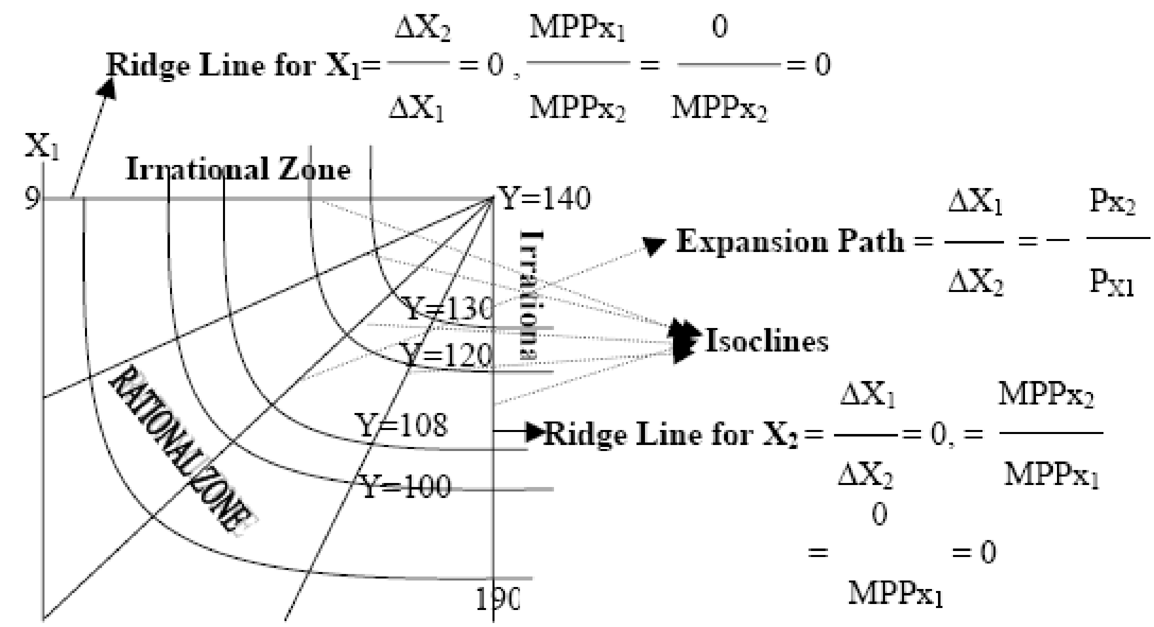
viii) Isoclines, Expansion Path and Profit Maximization: Isoclines are lines or curves that pass through points of equal marginal rates of substitution on an isoquant map. That is, a particular isocline will pass through all isoquants at

points where the isoquants have a specified slope. There are as many different isoclines as there are different slopes or marginal rates of substitution on an isoquant. The expansion path is also an isocline that connects the least cost combinations of inputs for all yield levels. On expansion path, the marginal rate of substitution must equal the input price ratio:

$$\textbf{Expansion Path } (\text{MRS}_{X2X1}) = \frac{\Delta X_1}{\Delta X_2} = -\frac{Px_2}{P_{X1}}$$

Ridge lines represent the limits of the economic relevance, the boundaries beyond which the isocline and isoquant maps have no economic meaning. The horizontal ridge line represents the points where $MPPx_1$ is zero and the vertical line represents the points where $MPPx_2$ is zero.

On the ridge line for X_1, MPP_{X1} is zero, and tangent to the isoquant which is vertical having no defined slope. On the ridge line for X_2, MPP_{X2} is zero and the isoquant has a zero slope and thus MRS = 0. Ridge lines are so named



Y= 45

7

Fig. 11.6 Isoclines and Expansion P....

The slope of the isoquant was shown to be: MRS of X_2 for $X_1 = -\frac{MPPx_2}{MPPx_1}$

because they trace the high points up the side of the production surface, much like mountain ridges that rise to the peak of the mountain. Ridge lines represent the points of maximum output from each input, given a fixed amount of the other input. When $X_1 = 1$, output can be increased by adding X_2 upto the amount devoted by the ridge line (7 units). At that point, output from X_2 is a maximum given one unit of X_1 and MPP $_{X2}$ is zero. Past $X_2 = 7$, MPP $_{X2}$ is negative while MPP X_1 is positive; the inputs have an opposite effect on output and are no longer substitutes. Thus, the ridge lines denote the limits of substitution. Outside

the ridge lines, the inputs do not substitute in an economically meaningful way. Output is maximum (140) where the ridge lines, and all other isoclines, converge. For 140 units of output, the least cost and only possible combination is 9 units of X_1 and 7 units of $X_{2.}$

ix) Expansion Path and Profit Maximization

The expansion path traces out the least cost combination of inputs for every possible output level. The question now arises; which output level is the most profitable? Conceptually, this question is answered by proceeding out the expansion path that is increasing output until the value of the product added by increasing the two inputs along the expansion path is equal to the combined cost of the added amount of two inputs. Viewed from the input side, this is same as saying that the VMP of each input must equal the unit price of that input; viewed from the output side, it is the same as saying the marginal cost must equal marginal revenue. Thus, while all points on an expansion path represent least cost combinations, only one point represents the maximum output level. For one output and two variable inputs, the equation is:

Profit = Py Y - $Px_1 X_1$ - $Px_2 X_2$ – TFC.

Maximizing this function with respect to the variable inputs gives two equations in two unknowns.

The above two equations can be written as: $VMPx_1 = Px_1$ (or) $VMPx_2 = Px_2$. Thus, the profit-maximizing criterion requires that the marginal earnings of each input must be equal to its cost

$Y = 18X_1 - X_1^2 + 14X_2 - X_2^2$

The marginal physical products of X_1 and X_2 are multiplied by Py = Rs. 0.65. Then, $VMPx_1 = (18 - 2X_1)\ 0.65$; $VMPx_2 = (14 - 2X_2)\ 0.65$

Equating VMP's with the input prices of Rs.9 and Rs.7 for X_1 and X_2 respectively, we get, $(18 - 2X_1)\ 0.65 = 9$; $(14 - 2X_2)\ 0.65 = 7$

The solutions are 2.08 for X_1 and 1.6 for X_2. Substituting these values in the production function predicts a value of Y equal to 53 units.

Then, Profit = 0.65 (53) - 9 (2.08) - 7 (1.60) - TFC= Rs. 4.53 - TFC

The optimum criterion for two variable inputs is often expressed as:

$$\frac{VMPx_1}{Px_1} = 1 \ ; \ \frac{VMPx_2}{Px_2} = 1 \text{ or because both equal 1, } \frac{VMPx1}{Px_1} = \frac{VMPx2}{Px_2} = 1$$

All variable inputs must be earning as much as they cost on the margin. Rewriting in a different form, yields :

$$MPPx_1 = \frac{Px_1}{Py} \ ; \ \frac{MPPx_1}{Px_1} = \frac{1}{Py} = \frac{1}{MC}, \text{ Since Py =MC}$$

$$MPPx_{2z} = \frac{Px_2}{Py} \ ; \ \frac{MPPx_2}{Px_2} = \frac{1}{Py} = \frac{1}{MC}$$

The above expression is the same as MR = MC under perfect competition.

B. ECONOMIES OF SCALE

The scale of production influences the cost of production. In general, larger the scale of production, the lower is the average cost of production. The term ‘economies’ means ‘advantages’ and the term ‘scale’, here, means ‘large-scale production’. Thus, economies of scale refer to the advantages of large-scale production. Economies of scale can be categorized into i) internal economies and (ii) external economies of scale.

i) Internal Economies

Internal economies are those economies in production (those reductions in production costs), which occur in the farm (firm) itself when it expands its output or enlarges its scale of production. Internal economics are those advantages that are exclusively available to a particular firm, as a result of its own expansion in the scale of production. The internal economies are dependent on the resources of the individual house of business, on their organization and the efficiency of their management. Internal economies are of the following five

types: a) Technical economies, b) Managerial economies, c) Marketing economies, d) Financial Economies, and e) Risk bearing economies.

a) Technical Economies: A large-scale production unit can use large and modern or sophisticated machines so as to reduce production costs. A large establishment can prevent wastage by utilizing the by-products efficiently. Latest technologies can be used in larger units to reduce the cost of production (E.g.) A big vegetable oil mill can have a cattle feed industry and a big dairy unit.

b) Managerial Economies: These economies arise from the creation of special (separate) departments for different functions like production, maintenance, purchase, sales etc. In a small factory, a manager is a worker, organizer and salesman. Much of his time is wasted on things of little economic importance. In a big concern, such jobs can be allotted to junior employees and the manager can concentrate on jobs which bring more profits. Such kind of division of labour is possible in large units. Thus, the job can be done more efficiently and more economically in large units.

c) Marketing Economies: They arise from the purchase of materials and sale of goods. Large business firms have better bargaining advantages and are provided with a preferential treatment by the firms they deal with. They are able to secure freight concessions from railways and road transport firms, prompt delivery and careful attention from all dealers. A large firm can employ expert purchase managers and sales managers. In selling, it can cut down selling costs and in purchasing, it can have a wider choice.

d) Financial Economies: A big firm has better credit facilities and can borrow on more favourable terms. It encourages prospective investors with incentives and higher returns and therefore, its shares have a wider market. A big firm can issue its shares and debentures more easily than an unknown small firm.

e) Risk-Bearing Economies: A big firm can spread risks and can often eliminate them. It can diversify the output. It can also establish wider marketing net work for its products. If demand for any one of its products slackens in any one market, it may increase it in other markets. Thus, it can reduce the risk of fluctuations in the demand for its product.

ii) External Economies

External economies are those economies, which are available to each member firm as a result of the expansion of the industry as a whole. Expansion of industry may lead to the availability of new and cheaper raw materials, machineries and to the use of superior technical knowledge. External economies are advantages available to all the firms. For instance, construction of a new railway line benefits all firms set up in that locality and not to any particular firm alone. Various types of external economies are given below:

a) Economies of concentration: These economies arise from the availability of skilled workers, the provision of better transport and credit facilities, benefits from subsidiary units and so on.

Scattered firms cannot enjoy such economies. Concentration of firms enables the transport system to reduce the cost.

b) Economies of Information: All big-sized units can join together to publish trade journals and also to set up research and development facilities, which would benefit all firms.

c) Economies of Disintegration: When an industry grows, it becomes possible to split up some of the processes which are taken up by specialist firms. This may be beneficial to all the firms.

iii) Diseconomies of Scale

Large-scale production also has some disadvantages, which are known as diseconomies of scale. They are as follows: a) If the growth of the firm expands beyond the optimum limit, it will become unwieldy. As a result, the management becomes inefficient. (b) There may be frequent breakdown of machines due to poor maintenance. (c) There may be indiscipline among labourers and this would result in frequent strikes and lock outs. (d) Some times the over production exceeds demand and causes depression and unemployment. (e) The average cost of production will be more.

C. ECONOMIES OF SIZE

A study on economies of size would be useful to assess the optimum size of the plant. The collection of all durable assets owned by a farm is called the plant and this term, therefore, includes land, machinery, buildings and other durable assets found on farms. An increase in any one of these durable assets would increase plant size. The long run average cost curve has the same shape as the short run ATC curve. (But, long run cost has no fixed cost). When the firm is small, expansion of output usually increases efficiency, and average costs per unit of output will fall. The reasons for this decrease include specialization of labour and capital.

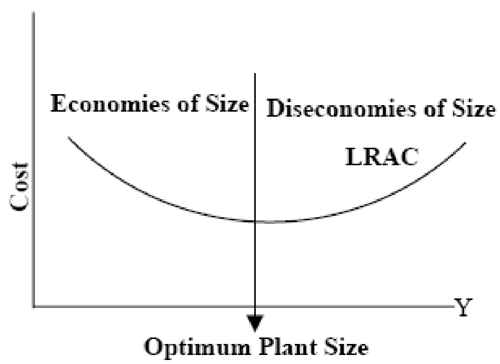


Fig.11.7 Economies of Size

As the size of the businessincreases, the manager may be ableto purchase inputs at a discount,thereby gaining market economies.Expansion of the firm enablesworkers to specialize and use moreadvanced or efficient technologies.Eventually, the long-run averagecurve will turn up; costs per unit ofoutput begin to increase as output isexpanded. As firm size increases, the manager encounters increasing difficulty in maintaining control of his organization, communications and coordination become more difficult, mistakes are both more frequent and more costly. As a result, costs rise. When LRAC are falling, the firm is experiencing economies of size. The minimum point on the LRAC curve indicates the optimum plant size. A plant of this size will produce the product at the lowest possible cost. Diseconomies of size occur where the LRAC curve is rising.

i) Relationship between Long run and Short-run Cost Curves

In the short - run, the farmer has a fixed plant-the number of acres, the buildings and the size and type of equipment are all fixed in amount. He can

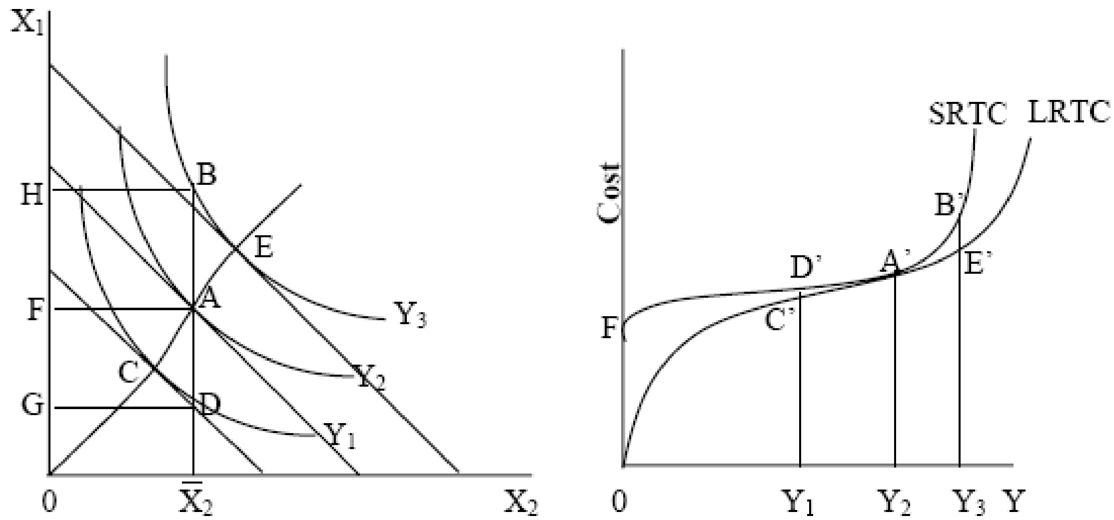


Fig 11.8 (a) *Fig.11.8 (b)*

Relationship between Short-Run and Long-Run Cost Curves

expand output in the short run only by changing the amount of variable input. This situation is depicted in Fig.11.8 (a). Plant size is fixed in the short run at X_2. That particular plant size, X_2 will produce output Y2 at the least average total cost when combined with OF amount of X1, the variable input. To produce outputs other than Y2 in the short run, the manager must vary the amount of X1 and by so doing, restrict input use to the combinations represented by the vertical line above X_2. For example, to produce output Y1 in the short run, the manager would use OG of X1 with the fixed plant, X_2 and to produce output Y3 in the short run the manager would use OH of X1 with X_2.

The combination of inputs at point A represents the least cost combination for the production of Y_2 in the long run, as do the combinations of by C for Y_1 and E for Y_3. Thus, the combination of inputs at D, OG of X_1 and X_2, necessarily costs more than the combination of inputs at point C on the long-run expansion path. Similarly, the combination at point B costs more than the combination at E. As a result, total costs in the short run along the line DAB will be higher than total costs in the long-run along the segment CAE on the long-run expansion path. The exception will occur at point A, where short-run and long - run costs will be equal.

This argument is also indicated in Fig. 11.8 (b). Fixed costs of amount OF are associated with X_2 amount of X_2. There are no fixed costs in the long-run. Short-run total costs, SRTC, increase with output but remain above long-run total costs, LRTC, until output Y_2 is reached. Point A' on LRTC and SRTC represents the cost of input combination at A in Fig.11.8 (a). At A' the two cost curves are tangent. At output levels greater than Y_2, SRTC increases more rapidly than LRTC. The costs of the input combinations C, D, E and B in

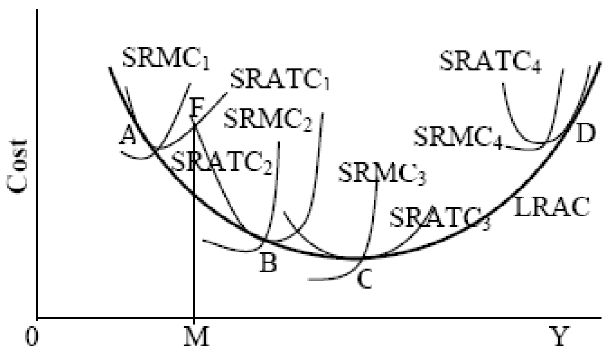


Fig.11.9 Long-Run Average Cost for Several Plan Sizes

Fig.11.8 (a) are represented by C', D',E' and B' in Fig.11.8 (b). In Figure11.9, the relationship between long-runaverage cost curve and several plantsizes is shown. $SRATC_1$ and $SRMC_1$are the average and marginal costs forplant size 1. $SRMC_2$ and $SRATC_2$ arethe marginal and average costs for alarger plant size, plant size 2. Eachplant size represents a set of durable

inputs fixed at a certain level. Many plant sizes exist between 1 and 2, but to avoid clutter, their cost curves are not shown. Plant size 2 produces all outputs larger than (to the right of) the amount OM more efficiently than does plant size 1. As output increases further, plant size 3 becomes more efficient than 2. For plant sizes larger than 3, expansion of output is obtained only at increased cost per unit. As explained, the LRAC will be tangent to each of the SRAC curves. Because of this, the LRAC is called *Envelope Curve*. To the left of C, the LRAC curve is tangent to short - run curves to the left of the latter's minimum. At C, both long-run and short-run costs attain a minimum. Therefore, plant 3 represents the optimum plant size. To the right of C, LRAC is tangent to the short-run curves to the right of the latter's minimum.

The LRAC curve depicts the minimum average cost for each output level and thereby determines the most efficient plant size for each output level. Plant size 1 is most efficient in the production of the output corresponding to A, plant size 2 for B, plant size 3 for C, etc. Thus, the LRAC is the *envelope curve* that is tangent to each SRATC curve at the output for which that plant size is most efficient in the long-run.

D. RETURNS TO SCALE

Returns to scale measures the change in output resulting form a proportionate change in all inputs. It describes the technical economies of scale and is a long-run concept, when none of the inputs is fixed. Returns to scale in increasing, constant or decreasing depending on whether a proportionate increase in all the inputs increases the output by a greater, same or smaller proportion. If the proportionate change in output is lesser than the proportionate change in inputs, diseconomies of scale result. If the change in output is equal to the proportionate change in inputs, constant returns to scale exist. If the change in output is greater than the proportionate change in inputs, economies of scale exist. This concept can be expressed with an homogeneous production function $Y = f(X_1, X_2,...,Xm)$ where, Y is output and X_1, X_2, ...,Xm are inputs used in the production process. Let K denote the amount by which each input will be changed $(1<K)$, i.e., K is any positive real number. The returns to scale will be defined by n where,

$$YK^n = f(KX_1, KX_2, . . . , KXm)$$

The factor Kn represents the change in output when all inputs are changed by the factor K. For example, if n equals one, the change in output is equal to the changes in the inputs and the returns to scale are constant. If n is greater than one, the change in output exceeds the proportionate change in all the inputs and returns to scale are increasing. Conversely, if n is less than one, the returns to scale are decreasing. In case of constant returns to scale, the distance between successive isoquants is constant, i.e., AB=BC=CD (Fig. 11.10 (a)). The distance goes on widening between isoquants when diminishing returns operate, i.e., $AB<BC<CD$ (Fig.11.10 (b)). Finally, in case of increasing returns to scale, the distance between the successive isoquants becomes smaller and smaller as we move away from the origin on the isoquant map i.e., $AB>BC>CD$ (Fig.11.10 (c)).

Returns to scale must be measured along a scale line, that is a straight line, passing through the origin. Proportionate input changes are possible only on such a line or ray. Thus, economies (diseconomies) of size are the same as economies (diseconomies) of scale only when the long long-run expansion path is a straight line passing through the origin. In most agricultural production situations, input proportions representing least cost combinations vary with the level of output. Therefore, strict interpretations of scale concepts are probably not of great value in agriculture.

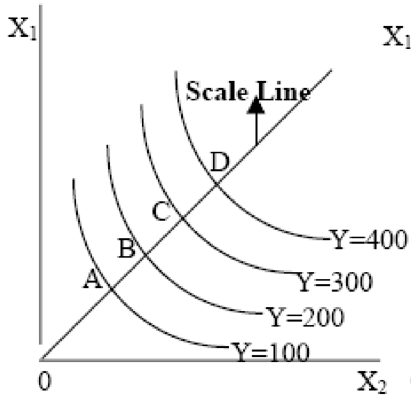


Fig.11.10 (a) Constant Return to Scale

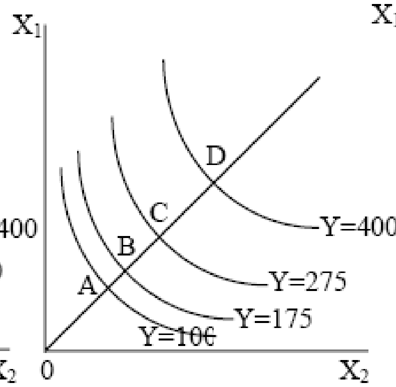


Fig.11.10 (b) Decreasing Return to Scale

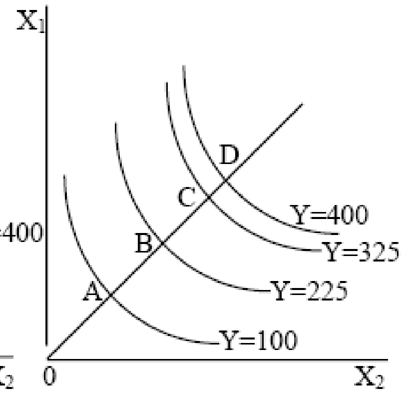


Fig.11.10 (c) Increasing Return to Scale

E. ELASTICITY OF FACTOR SUBSTITUTION

In the factor-factor relationship, the elasticity of factor substitution is defined as the percentage change in one input, X_1, in response to the percentage change in other input, X_2. Thus,

$$Es = \frac{\left\{\frac{\Delta X_1}{X_1}\right\}}{\frac{\Delta X_2}{X_2}} = \frac{\Delta X_1 X_2}{\Delta X_2 X_1} = (\text{MRS of } X_2 \text{ for } X_1)\left\{\frac{X_2}{X_1}\right\}$$

Elasticity of factor substitution is negative for inputs that are technical substitutes because the slopes of isoquants are negative; as one input increases, the other input decreases on the same isoquant. Elasticity of substitution for inputs that are technical complements is zero because MRS is zero. The elasticity of substitution measures and indicates the rate at which the slope of an isoquant changes. This is useful because it is expressed independent of unit of measurement.

Problem: Nitrogenous and Phosphorus fertilizer combination necessary to produce 2000 kgs of paddy is given in the following table. It shows how and to what extent nitrogen could be substituted for phosphorus fertilizer.

Table 11.2 Nitrogen and Phosphorus combinations Necessary to Produce Two Tonnes of Paddy

Combination Number	Nitrogen (X_1)(Kgs)	ΔX_1	Phosphorus (X_2)(Kgs)	ΔX_2	MRS of X_2 for X_1 =(ΔX_1 / ΔX_2)
1	52	-	11	-	-
2	44	- 8	12	1	- 8.00
3	38	- 6	14	2	- 3.00
4	33*	- 5	18*	4	- 1.25
5	30	- 3	23	5	- 0.60
6	28	- 2	29	6	- 0.33

(*Least cost combination of X_1 and X_2 inputs. If Px_1 and Px_2 are Rs. 6 and Rs.7.50 per kg respectively, the price ratio would be Px_2 / Px_1 = 7.50 / 6.00= 1.25). Thus, as shown in the table, we find that it is from combination 4, the least cost combination of 33 kgs of N and 18 kgs of P_2O_5 can be obtained, because it is at this point, the price ratio is equal to MRS or RTS, i.e.,

$$-\frac{\Delta X_1}{\Delta X_2} = \frac{Px_2}{Px_1}$$