

Lecture No. 8

Factor - Factor relationship - Principle of substitution - isoquant, isocline

In factor-product relationship, we studied the situation where only one input is varied and all other variables are held constant. But in most real world situations, two or more inputs are often varied simultaneously. So a farmer must choose the particular combination of inputs which would minimize the cost for a given output level. Thus, the main objective here is minimization of cost at a given level of output. When two or more inputs are variables, a given amount of output may be produced in more than one way, i.e., there is a possibility of substituting one factor (X_1) for another (X_2) as product level (Y) is held constant. The objectives of factor-factor relationship are i) minimization of cost at a given level of output and ii) optimization of output to the fixed factors through alternative resource-use combinations. In this case, the production function is given by $Y = f(X_1, X_2, X_3, \dots, X_n)$, i.e., the production depends on the amounts of X_1 and X_2 while the other inputs are held constant.

A. ISOQUANT (OR) ISO PRODUCT CURVE

An isoquant is defined as the locus of all combinations of inputs, X_1 and X_2 , for obtaining a given level of output, say, $Y(0)$. Mathematically, such an isoquant is written as: $X_1 = f(X_2, Y(0))$ or $X_2 = f(X_1, Y(0))$. Some of the important types of isoquants depending upon the degree of substitutability between the inputs, are given in figures below:

i) Linear Isoquant: In this case, two inputs substitute at constant rates. Labour input supplied by different persons substitutes at a constant rate. Ammonium sulphate containing 20.6 per cent nitrogen and urea containing 46 per cent nitrogen would substitute for each other at a constant rate. The decision rule is simple, i.e., use either of the two factors of production depending on their relative prices. The rate at which one factor (X_1) is substituted for one unit increase in another factor (X_2) at a given level of output is called Marginal Rate of Substitution (MRS). The marginal rate of substitution of X_2 for X_1 is denoted by: 187

In linear isoquant, the rate at which these two inputs can be substituted at a given level of output is constant regardless of the level of the two inputs used.

$$MRS_{x_2 x_1} = \frac{\Delta X_{11}}{\Delta X_{21}} = \frac{\Delta X_{12}}{\Delta X_{22}} = \dots = \frac{\Delta X_{1n}}{\Delta X_{2n}}$$

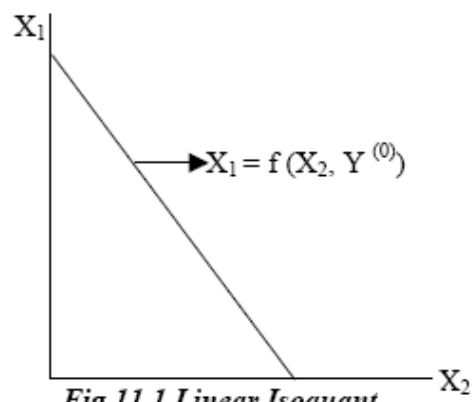


Fig.11.1 Linear Isoquant

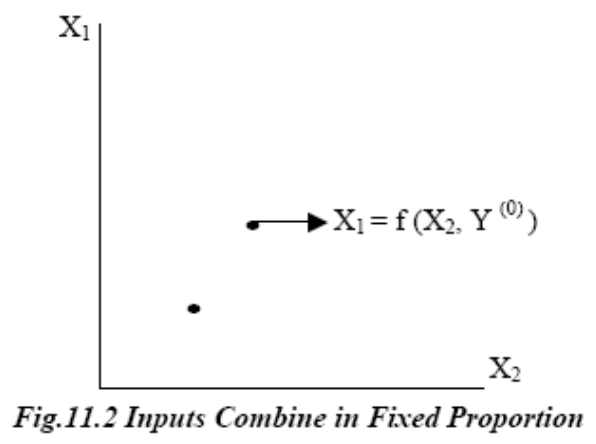


Fig.11.2 Inputs Combine in Fixed Proportion

ii) Fixed Proportion Combination of Inputs (Perfect Complements): Inputs that increase output only when combined in fixed proportions are called technical complements. Only one exact combination of inputs will produce the specified output. A tractor and a driver may serve as a fairly good example. Here, there is no economic problem in decision-making because there exists no alternative choice.

iii) Varying (Decreasing) Rate of Substitution: The amount of one input (X_1) required to be substituted for by one unit of another input (X_2) at a given level of production decreases. This is known as decreasing rate of substitution. Thus, decreasing rate of substitution means that every subsequent increase in the use of one factor replaces less and less of the other

Each point on the isoquant is the maximum output that can be achieved with the corresponding input combination. Isoquants are convex to the origin Fig.11.3. Two isoquants do not intersect each other.

iv) High contours represent higher output levels: Isoquant map indicates the shape of the production surface, which again indicates the nature of output response to the inputs. An isoquant which is far away from the origin represents higher level of output than an isoquant which is closer to the origin.

v) Marginal Rate of Input (Factor) substitutions (or) Rate of Technical Substitution (RTS):

Marginal Rate of Substitution of X_2 for X_1 ($MRS_{X_2X_1}$) is defined as the amount by which X_1 must be decreased to maintain output at a

constant level when X_2 is increased by one unit. Between the two points ($X_2 = 2, X_1 = 10$) and ($X_2 = 3, X_1 = 5$), the MRS of X_2 for X_1 is,

The MRS is negative, because the isoquant slopes downward and to the right;

$$\Delta X_1 = 5 - 10 = -5$$

$$MRS_{X_2X_1} = \frac{\Delta X_1}{\Delta X_2} = \frac{-5}{1} = -5$$

$$\Delta X_2 = 3 - 2 = 1$$

that is, the isoquant has a negative slope.

Table 11.1 Decreasing Rate of Substitution

Variable Input (X ₁)	Variable Input (X ₂)	ΔX ₁	ΔX ₂	MRS = $\frac{\Delta X_1}{\Delta X_2}$
23	0	-	-	-
16	1	7	1	-7
10	2	6	1	-6
5	3	5	1	-5
1	4	4	1	-4
0	5	1	1	-1

vi) **Isocost line:** Isocost line determines all combinations of the two inputs that cost the same amount. Each point on the isocost line represents a combination of

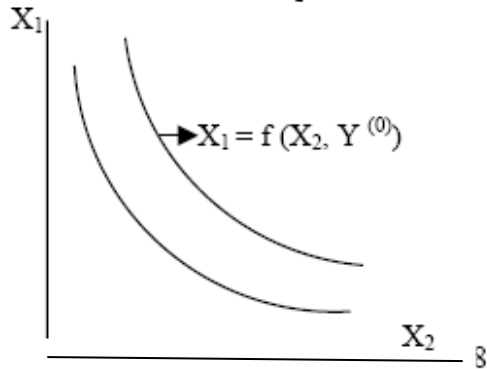


Fig.11.3 Convex Isoquant.

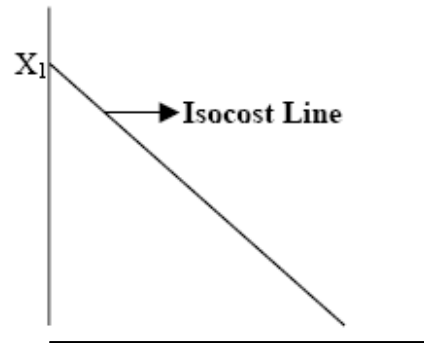


Fig.11.4 Isocost Line

inputs that can be purchased with the same outlay of funds (Fig.11.4). Under constant price situation, each possible total outlay has a different isocost line. As total outlay increases, isocost line moves higher and higher and moves farther away from the origin. Changes in the input prices will change the slope of isocost line as the slope indicates the ratio of input prices. A decrease in the input price means that more of that input can be purchased with the same total variable cost; an increase means that less can be purchased.

Total Outlay = Rs.36; Price of X₁ = Rs. 4; Price of X₂ = Rs.3; When X₁ = 0, X₂ = 12; When X₂ = 0, X₁ = 9.

The equation of the isocost line can be found by solving the TVC equation

for X₁ as an explicit function of X₂. $P_{X_1}X_1 = TVC - P_{X_2}X_2$ and

$$X_1 = \frac{TVC}{P_{X_1}} - \frac{P_{X_2}}{P_{X_1}} X_2$$