

Lecture No.6

Cost concepts & interrelations - Optimum level of input use and optimum production

B. COSTS

Production costs play an important role in the decisions of the farmers. Explicitly or implicitly, most of the producers keep in mind the cost of producing additional units of output. In general, at given level of prices, a farmer can increase his farm income in two ways, i.e., i) by increasing production and / or ii) by reducing the cost of production. Since cost minimization is an individual skill, degree of success in this direction directly adds to the profits of the farm.

Costs refer to the money value of effort extended or sacrifice made in producing an article or rendering a service or achieving a specific purpose. Costs, thus, are the expenses incurred in organizing and carrying out the production process. They include outlays of funds for inputs and services used in production. Money value of all inputs used in the production process is termed as the total cost. If the inputs used are represented by X_1, X_2, \dots, X_n and the

respective prices by $P_{X_1}, P_{X_2}, \dots, P_{X_n}$, then the total cost (TC) can be expressed as: $TC = P_{X_1} \cdot X_1 + P_{X_2} \cdot X_2 + \dots + P_{X_n} \cdot X_n$.

i) Total costs: The total cost comprises of two components, i.e., fixed and variable costs. Costs of fixed inputs are called fixed costs, while costs of variable inputs are called variable costs. TVC and TC increase as output increases. Fixed costs do not change in magnitude as the amount of output of the production process changes and are incurred even when production is not undertaken, i.e., fixed costs are independent of output. Land revenue, taxes, contractual payments such as rent and interest on capital for the use of fixed resources, and the value of services from fixed resources all represent fixed costs. In farming, cash fixed costs include land taxes, rent, insurance premium, etc. Non-cash fixed costs include depreciation of building, machineries and equipments caused by the passing of time, interest on capital investment, charges for family labour and charges for management. Variable costs constitute the outlay of funds that are a function of output in a given production period, i.e., they vary with the level of output. The outlays of funds on seed, fertilizer, insecticide, casual labour, fuel and oil, feeds, etc, are a few examples of variable costs. Total fixed costs and total variable costs are denoted by TFC and TVC respectively. In short run, total costs include fixed and variable costs. When no variable input is used, $TC = TFC$. The TC curve at any point is equal to the vertical addition of TFC and TVC. In the long run, all costs are considered variable costs because all inputs are variable. Conventionally, total cost (TC) is thought of as a function of output (Y), rather than that of inputs, i.e., $TC = f(Y)$. Such a cost curve is illustrated with TC on the vertical axis and Y (output)

on the horizontal axis. The total cost curve is similar to the production curve, when the physical units of X (input) have been replaced with the corresponding cost ($P_x X$). Hence, the shape of the TC curve, like that of the TVC curve, depends upon the production function. In symbolic notation, TC can be written as

$$TC = TFC + TVC = TFC + P_x X, \text{ where } TVC = P_x X \text{ (ii)}$$

Average Fixed Cost, Average Variable Cost and Average Total Cost

a) Average Fixed Costs (AFC): Average fixed costs, (AFC) are computed by dividing total fixed costs by are amount of output. AFC varies depending on the amount of production.

$$AFC = TFC / Y$$

b) Average variable cost, AVC: It is computed by dividing total variable cost by the amount of output. AVC varies depending on the amount of production. The shape of the AVC curve depends upon the shape of the production function while AFC always has the same shape regardless of the production function.

c) Average Variable Cost and Average Physical Product: Average variable cost is inversely related to average physical product. When APP is increasing, AVC is decreasing. When APP is at its maximum, AVC attains a minimum value. When APP is decreasing, AVC is increasing. Thus, for a production function, APP measures the efficiency of the variable input, while AVC provides the same measure for cost curves. When AVC is decreasing, the efficiency of the variable input is increasing; efficiency is at a maximum level when AVC is a minimum and is decreasing when AVC is increasing. The relationship algebraically is as follows:

$$AVC = \frac{TVC}{Y} = \frac{P_x X}{Y} = P_x \frac{X}{Y} = P_x \cdot APP$$

, because $\frac{X}{Y} = APP$

d) Average Total Costs, ATC: It can be computed in two ways. Total costs can be divided by output or AFC and AVC can be added.

$$ATC = TC / Y. \text{ (or) } ATC = AFC + AVC.$$

e) Nature and Relationship between Cost Curves: The shape of the ATC curve depends upon the shape of the production function. ATC decreases as output increases from zero, attains a minimum, and increases thereafter. ATC is referred to as unit cost of production, i.e., the cost of producing one unit of output. The initial decrease in ATC is caused by the spreading of fixed costs

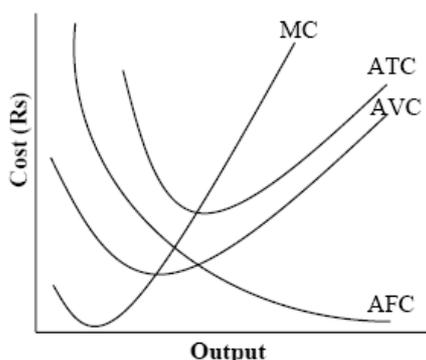


Fig.10.7 Average and Marginal Cost Curves

among an increasing number of units of output and the increasing efficiency with which the variable units is used (as indicated by the decreasing AVC curve). As output increases further, AVC attains a minimum and begins to increase; when these increases in AVC can no longer be offset by decreases in AFC, ATC begins to rise. AVC reaches its lowest point earlier than ATC.

iii) Marginal Cost, MC: It is defined as the change in total cost per unit increase in output. It is the cost of producing an additional unit of [output. MC](#) is computed by dividing the change in total costs, OTC, by the corresponding change in output, ΔY , i.e., $MC = OTC / \Delta Y$. By definition, the only change possible in total costs is the change in Δ variable cost, because fixed cost does not vary as output varies. Thus, $OTC = ATVC$. Therefore, MC could also be computed by dividing the change in total variable cost by the change in output. Geometrically, MC is the slope of the TC and the TVC curve. The shape of the MC curve is in an inverse relationship to that of MPP. For lower levels of output, MC is decreasing while MPP is increasing. Algebraically, the

$$MC = \frac{\Delta TC}{\Delta Y} = \frac{\Delta TVC}{\Delta Y} = \frac{P_x (\Delta X)}{\Delta Y} = P_x \cdot \frac{\Delta X}{\Delta Y} = \frac{P_x}{MPP}$$

MC and AVC are equal, where MPP is equal to APP. For lower output levels, MC is less than AVC and ATC and for higher output levels, MC is greater than AVC and ATC. As long as there is some fixed costs, MC crosses ATC at an output greater than the output at which AVC is at the minimum and MC is equal to ATC at the latter's minimum [point. MC](#) curve will intersect the AVC and ATC curves at their lowest point from below.

Costs need be computed and graphed for input and output amounts only in stages I and II of the production function; stage III is an area in which no rational manager would produce. Stage II begins at the point where $MC=AVC$ and continues to the point where output is a maximum.

$$TC = 100 + 8Y - 0.4Y^2 + 0.02Y^3$$

$$TFC = 100$$

$$TVC = 8Y - 0.4Y^2 + 0.02Y^3 \quad AVC =$$

$$TVC = 8 - 0.4Y + 0.02Y^2$$

$$AFC = \frac{TFC}{Y} = 100$$

$$ATC = AFC + AVC = 100Y^{-1} + 8 - 0.4Y + 0.02Y^2$$

$$MC = \frac{dTVC}{dY} = 8 - 0.8Y + 0.06Y^2$$

iv) Methods of Determining the Optimum Level

The problem here is to determine the most profitable point of operation for an enterprise in the short-run. This can be done by determining either the most profitable amount of input or the most profitable level of output. As the production function relates the input to output in a unique manner in stage II, either method results ultimately the same answer. In economic terminology, the “most profitable” amount

can be called the “optimum” amount.

a) Determining the Optimum using Total Value Product and Total Costs:

Total Value Product, TVP, is the total value of the production of an enterprise. $TVP = P_y \cdot Y$, where P_y is the price per unit of the output and Y is the amount of output at any level of input X . Total value product minus total cost is the profit which is also called net returns or net revenue. As output increases, profit increases and reach a maximum of Rs.30 at 8 units of input and 28 units of output as could be seen from the table 10.4. Maximum profit from an enterprise does not necessarily occur where output is at its maximum. Output reaches a maximum of 29 units at 9 units of input. Therefore, the point of maximum yield is not necessarily the same as the point of maximum profit. $Profit = TVP - TC = TVP - TVC - TFC = P_y \cdot Y - P_x X - TFC$.

b) Determining the Optimum Amount of Input

The criterion for determining the optimum amount of input is derived from the slopes of total value product and total cost curves, when those curves are plotted as functions of the input, X . First, consider the profit equation as the function of input.

$$Profit = P_y \cdot f(X) - P_x X - TFC, \text{ where, } Y = f(X).$$

In order to maximize this function with respect to the variable input, the first derivative is set to zero as follows:

$$\begin{aligned} \frac{d(Profit)}{dX} &= P_y \frac{dY}{dX} - P_x = 0 \\ &= P_y \cdot MPP - P_x = 0 \\ P_y \cdot MPP &= P_x, \text{ i.e., } VMP = P_x. \end{aligned}$$

But, the term $P_y \cdot MPP$ is the slope of TVP curve and is called the Value of Marginal Product (VMP). The term P_x is the slope of the total cost function. In pure competition, P_x will always be constant. Dividing both sides of P_y , we get,

$$MPP = \frac{P_x}{P_y}. \text{ So, another method of stating the marginal criterion is to say that}$$

the marginal product of variable input must equal the inverse ratio of prices (input - output price ratio).

c) Determining the Optimum Amount of Output

The marginal conditions for the maximization of profit as a function of output can be derived from the following profit function:

$$Profit = TR - TC = P_y \cdot Y - P_x \cdot X - TFC = P_y \cdot Y - P_x \cdot f^{-1}(Y) - TFC$$

where, the concept of the inverse production function must be used to express X as a function of Y. That is, $X = f^{-1}(Y)$ in stages I and II. Taking the derivatives of profit with respect to Y results in: d (profit)

$$\begin{aligned} \frac{d(\text{profit})}{dY} &= P_y - P_x \frac{dX}{dY} = 0 \\ &= P_y - \frac{P_x}{MPP} = 0 \\ P_y &= MC \cdot \text{Since } MC = \frac{P_x}{MPP} \end{aligned}$$

Therefore, $P_y = MC$ at the optimum output level. Differentiation of profit equation with respect to Y would give d (Profit)

$$\begin{aligned} \frac{d(\text{Profit})}{dY} &= \frac{dTR}{dY} - \frac{dTC}{dY} = 0 \\ \frac{dTR}{dY} &= \frac{dTC}{dY} \quad \text{i.e., } \underline{MR=MC} \end{aligned}$$

where, the change in TR with respect to Y is defined as Marginal Revenue, MR, while the change in TC with respect to Y is the Marginal Cost, MC. In pure competition $P_y = MR$.

d) Comparison of Input and Output Criteria: All methods of determining the most profitable level of output or input lead to comparable answers. The input criterion $VMP = P_x$ can be written as $P_y \cdot MPP = P_x$ (or)

$$P_y \cdot \frac{\Delta Y}{\Delta X} = P_x \cdot \text{That is, } P_y \cdot \Delta Y = P_x \cdot \Delta X$$

Table 10.4 Product-Cost Relationships

Input (Units)	Total Output (Units)	Average Product =Y/X (Units)	Marginal Product = $\Delta Y/\Delta X$ (Units)	Total Fixed Costs (TFC) (Rs)	Total Variable Cost (TVC) @ Rs.2/Unit	Total Cost= TFC+ TVC (Rs)	Average Fixed Cost (AFC) =TFC/Y	Average Variable Cost (AVC) = TVC/Y	Average Total Cost (ATC) = TC/Y	Marginal Cost (MC) = $\Delta TC/\Delta Y$	Marginal Revenue (MR) = $\Delta TR/\Delta Y$
0	0	0	0	10	0	10	0	0	0	-	-
1	2	2.00	2	10	2	12	5.00	1.00	6.00	1.00	2
2	5	2.50	3	10	4	14	2.00	0.80	2.80	0.67	2
3	9	3.00	4	10	6	16	1.11	0.67	1.78	0.50	2
4	14	3.50	5	10	8	18	0.71	0.57	1.28	0.40	2
5	19	3.80	5	10	10	20	0.53	0.53	1.06	0.40	2
6	23	3.83	4	10	12	22	0.43	0.52	0.95	0.50	2
7	26	3.71	3	10	14	24	0.38	0.54	0.92	0.67	2
8	28	3.50	2	10	16	26	0.36	0.57	0.93	1.00	2
9	29	3.22	1	10	18	28	0.34	0.62	0.96	2.00	2
10	29	2.90	0	10	20	30	0.34	0.69	1.03	-	2
11	28	2.55	-1	10	22	32	0.36	0.79	1.15	-	2
12	26	2.17	-2	10	24	34	0.38	0.92	1.30	-	2

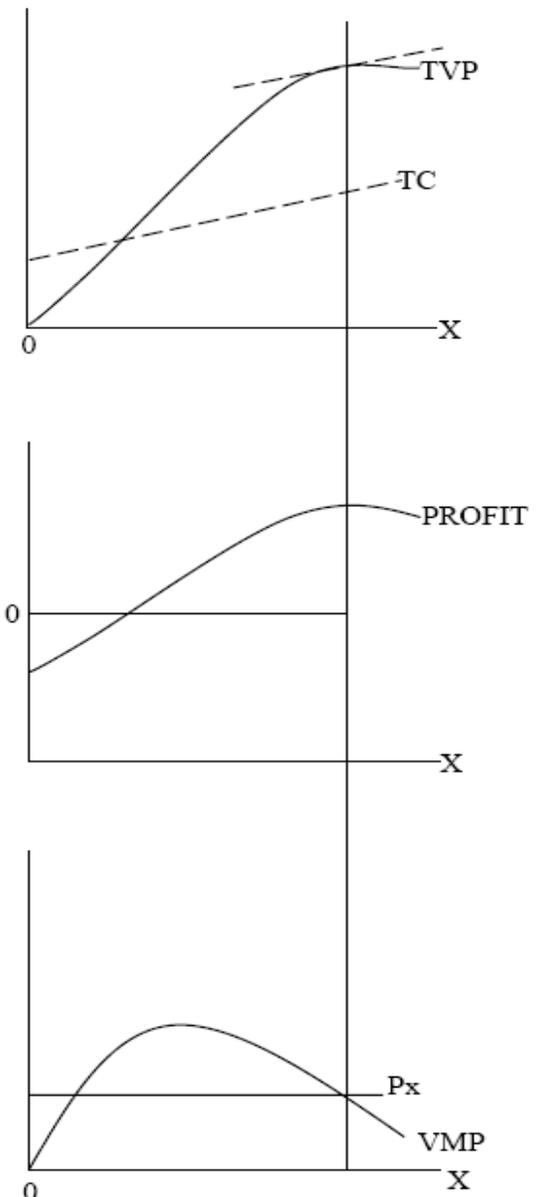


Fig.10.8 (a) Determining the Optimum Amounts of Input Using Total Value Product, Total Cost, Profit and Value of Marginal Product Curves.

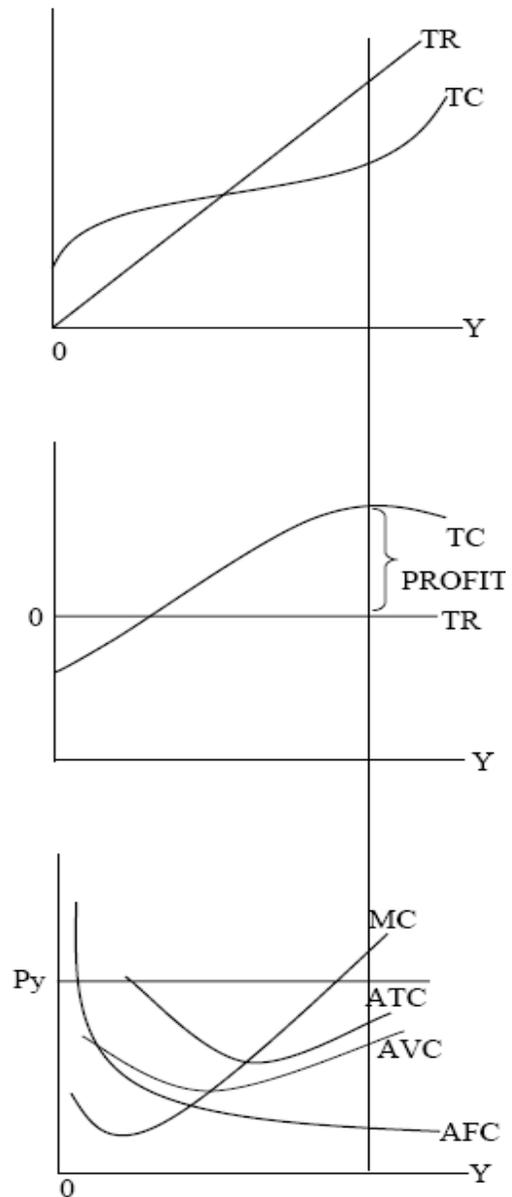


Fig. 10.8 (b) Determining the Optimum Amount of Output Using Cost and Revenue Curves

The expression, $P_y \cdot \Delta Y$ measures returns added by an increase in output while $P_x \cdot \Delta X$ measures the cost added by the increase in input. But from the output criterion, $P_y = MC$, the same added cost and added return can be derived.

$$P_y = MC = \frac{P_x \cdot \Delta X}{MPP \cdot \Delta Y} = P_x \frac{\Delta X}{\Delta Y}$$

$$P_y \cdot \Delta Y = P_x \cdot \Delta X$$

Thus, the two methods of determining the optimum levels are comparable.

*Table 10.5 Response of Paddy to Nitrogen:
Application of Profit Maximization Principle.*

Variable Input (X)	Output (Y)	Total Cost =TFC (Rs.10)+TVC @Rs.2/unit of input	Total Value Product = $P_y \cdot Y$ @Rs.2/unit of output	Profit = TVP-TC
0	0	10	0	-10
1	2	12	4	-8
2	5	14	10	-4
3	9	16	18	2
4	14	18	28	10
5	19	20	38	18
6	23	22	46	24
7	26	24	52	28
8	28	26	56	30
9	29	28	58	30
10	29	30	58	28
11	28	32	56	24
12	26	34	52	18